**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

**Ans.**

Mean (**μ**) = 45 minutes

Std (**σ** ) = 8 minutes

**X**= 60-10 =50 minutes

The work begins after 10 min, so the average time increase from

45min to 55min.

for normal distribution: - X ~N(45,8)

Z = (X-μ)/σ

= (50-45)/8

= 0.625

Probability that the service manager will not meet his demand will be

P(X>50) = P ( (X-μ)/σ > (50-45)/8 )

= P( Z > 0.625 )

= 0.2676 [ 1 – stats.norm.cdf(abs(Z\_Score)) ]

OR

In R Software

(1 -pnorm(50,45,8)) = 0.2659

1. 0.3875
2. **0.2676**
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and 44.

**False**

Around 70% of the data falls within one standard deviation of the mean (µ+σ= 38+6=44)

If more employees are older than 44, this will shift the*μ*  towards 44 with considering sd. Which is not possible as *μ* is given 38 with Sd 6.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**True**

Z=(X-µ)/ σ

Employees under 30 yrs of age P(X≤30) :

P(X≤30) =p (Z ≤ (30-38) /6)

= p (Z ≤ -1.33)

= 0.0918( by z table)

OR

Employees under 30 yrs of age P(X≤30) :

stats.norm.cdf (30,38,6)

Expected count=0.0918\*400= **36.72**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans.**

if X ∼ N (µ1, σ1^2) **And** Y ∼ N (µ2, σ2^2) are two independent random variables

then

X + Y ∼ N (µ1 + µ2, σ1^2 + σ2^2) **And** X − Y ∼ N (µ1 − µ2, σ1^2 + σ2^2)

Similarly, if Z = aX + bY i.e Z is linear combination of X and Y,

then

Z ∼ N (aµ1 + bµ2, a^2σ1^2 + b^2σ2^2)

Therefore

2X1~ N (2 u,4 σ^2)

X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4µ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

**Ans. Option D**

The Probability of getting value between a and b is 0.99

Probability outside the a and b area is 0.01 (i.e. 1- 0.99).

0.01/2 =0.005

The Probability towards left from a = -0.005

The Probability towards Right from b = +0.005

For Probability 0.005 the Z Value is -2.57 (by Z Table)

Z =(X- μ) / σ

X = Z \* σ + μ

Z(-0.005)\*20+100 = **151.4**

Z(+0.005)\*20+100 = **48.6**

OR

stats**.**norm**.**interval(0.99,100,20)

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. **48.5, 151.5**
5. 90.1, 109.9
6. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
7. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans.** Mean Profit is Rs 540 Million [ (5+7)=12\*45 = 540 ]

Variance of profits from two different divisions of a company = SD^2 = SD1^2 + SD2^2

Standard Deviation is Rs 225.0 Million [ SD = √(9)+(16) =5\*45 = 225.0

(stats.norm.interval(0.95,540,225)) = (99.00,990.00)

**So Profit of the company in Rupees(in Million) is between range [99.00, 990.00]**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans.** To compute 5 th percentile ,we use the formula **X = μ + Zσ**; wherein

from z table, **5 th percentile** = -1.645

X= 540+(-1.645) \* (225)

Therefore, 5th percentile of profit (in Million) **is 170.00**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans.**  **Using stats.norm.cdf(0,5,3) Probability of Division 1 making a loss P(X<0)**

**is 0.04780 and stats.norm.cdf (7,4) probability of Division 2 making a loss**

**P(X<0) is 0.04005.**

**Ans.is Division 1 has Larger Probability of Making Loss in a given year**